Particle migration and stability in yield-stress fluid internal flows

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Abstract

Particle stability and motion in a yield-stress fluid flow inside a 2D channel has been considered. Yield-stress fluids behave as solids (rigid/viscoelastic solid) when the imposed stress is small (less than the yield stress). Otherwise, they will flow. This special characteristic leads to the formation of an unyielded (plug) region in the middle of the channel, therefore, the yield-stress fluid can hold particles buoyant in the core unyielded region if the yield stress is large enough: static stability. An interesting question is that how large the yield stress should be to reach this static stability limit. We will report the critical yield stress (or in the non-dimensional space, the *yield number*) which marks this stability margin. Moreover, previous experimental observations have revealed that big particles will stick to the wall and will stay there. We will investigate this possibility in detail. Some interesting particle dynamics (e.g., inertial migration) can be observed in the sheared regions as well.

Introduction and preliminary results

This study focuses on the motion of particles in Poiseuille flow of a yield-stress fluid. Since the pioneering work of Segré & Silberberg (1961) in which the first observations were made of the particle migration, this problem has received significant attention. When it comes to the yield-stress fluids, the problem of particle migration and settling under the influence of gravity is very important for a large number of applications, such as pumping mortar/concrete and also in the oil & gas industries.

We will consider two rheological models to explain the behavior of the fluid: viscoplastic (Bingham model) and elastoviscoplastic (Saramito model (2007)) fluids,

$$\mathcal{W}i \, \overset{\triangledown}{\boldsymbol{\tau}} + \left(1 - \frac{\mathcal{B}}{\|\boldsymbol{\tau}_d\|}\right)_+ \boldsymbol{\tau} = (1-\beta)\,\dot{\boldsymbol{\gamma}}, \qquad (1)$$

where $\mathcal{W}i$ and $\mathcal{B}$ are the Weissenberg and Bingham numbers, which take the form, $\mathcal{W}i = \frac{\hat{\lambda}\hat{U}}{\hat{d}}$ & $\mathcal{B} = \frac{\hat{\tau}_Y \hat{d}}{\hat{\mu}\hat{U}}$, respectively. The extra stress tensor is represented by $\boldsymbol{\tau}$, the "$\triangledown$" sign shows the upper-convected derivative, β represents the ratio of the solvent viscosity to the total viscosity, $\dot{\boldsymbol{\gamma}}$ shows the rate of strain tensor, $\hat{\lambda}$ is the relaxation time of the fluid, $\hat{d}$ is the diameter of the particle, $\Delta\hat{\rho}$ is the density difference between the particle and the fluid, $\hat{g}$ is the gravitational acceleration, and $\hat{\tau}_Y$ is the yield stress of the fluid. For a Bingham fluid, $\mathcal{W}i$ and β are equal to zero. Moreover, the governing equation can be written as:

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\nabla p + \frac{1}{\mathcal{R}e}\left(\boldsymbol{\nabla}\cdot\boldsymbol{\tau} + \beta\,\boldsymbol{\nabla}\cdot\dot{\boldsymbol{\gamma}} - \frac{\mathcal{B}}{\mathcal{Y}}\frac{\rho}{1-\rho}\boldsymbol{e}_y\right), \qquad (2)$$

where $\mathcal{Y}$ is the yield number which represents the ratio of the yield stress to the buoyancy stress, $\mathcal{Y} = \frac{\hat{\tau}_Y}{\Delta\hat{\rho}\hat{g}\hat{d}}$, and ρ is the ratio between fluid and particle densities, $\hat{\rho}_f/\hat{\rho}_s$.

Different senarios can occur based on the value of the parameters and the initial position of the particle:

(i) If the particle is totally inside the sheared region, it will migrate towards its stable position. However, the stable position is a function of $\mathcal{R}e$, $\mathcal{W}i$, $\mathcal{B}$, and $\mathcal{Y}$. It has been observed by Merkak et al. (2008) that if the particles are close enough to the pipe wall, they will stick to the wall.

(ii) If the particle starts completely inside the core plug region, then it may just translate with the plug velocity, if the buoyancy stress is not large compare to the yield stress. In other words, if $\mathcal{Y} > \mathcal{Y}_c$, then yield stress will hold the particle inside the plug region.

(iii) If the particle is partially in the plug region and partially in the sheared region, then it may hold there or will break the plug and go into the sheared region based on the value of the set of parameters.

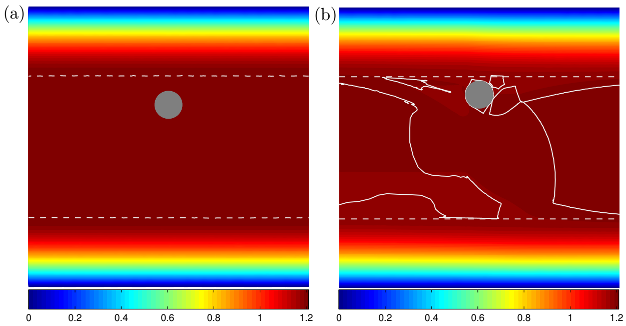

Figure 1: Velocity contours ($\mathcal{R}e = 1$, $\mathcal{W}i = 0$): (a) $\mathcal{B} = 1, \mathcal{Y} = 1.1$, (b) $\mathcal{B} = 1, \mathcal{Y} = 0.07$.

All three cases above are considered in the present study. For example, in the case (ii), the aim is to find the value of $\mathcal{Y}_c$: if $\mathcal{Y} > \mathcal{Y}_c$ then it means that the yield stress is large enough to hold the particle inside the core plug region (see Fig. 1a), otherwise, the particle will break the plug and starts to sediment (see Fig. 1b). The critical yield number has been reported by Chaparian & Frigaard (2017) for a 2D cylinder inside a quiescent fluid. However, in the present problem because of the background shear stress, $\mathcal{Y}_c$ increases dramatically. This is important since it can significantly change the homogeneity of the suspension.